1. Introduction

Due to the demand of the market, an increasing number of producers develop methods to minimize the time that elapses between ordering and receiving the finished product by the customer, especially by introducing Make-To-Order production (MTO), based mostly on orders from sales centres or even directly from customers. While the idea of production of this type is extremely simple (a company is supposed to manufacture exactly what was ordered), the matter of execution is a bit more difficult. Firstly, the company should have the proper production capacity at disposal when the demand comes up, secondly, all the components required for a particular order execution should appear on the main assembly line on time, thirdly, the company must be able to flexibly manage production to provide effective utilization of all available resources and to ensure timely delivery of the final product while, of course, maintaining a high level of quality.

Products with unique features selected by the customer are now the most widely produced in production lines [1, 2]. Continuous linear mixed-model assembly systems arose through evolution from linear systems prepared for the assembly of one product (single-model assembly lines, Figure 1). It is a solution that allows standardization of processes and smooth flow of products during the assembly. Based on the principles of economies of scale, the use of assembly lines allows reducing unit production costs – after introducing the assembly line by Ford, Model T’s price in 10 years decreased from almost $900 to about $290. Introducing assembly lines was the largest so far and one of the few such significant revolutions in assembly processes. The second most frequently given revolution is the rise of TPS (Toyota Production System, or Thinking People System [3]), with the proviso, that it didn’t provide such significant improvement in the performance of assembly processes.

In modern production systems, on the lines various products (models) or different varieties of the same product (versions), differing in e.g. version of the body, components and accessories, and though differing also in labour-consumption are manufactured. One of the problems that arise in mixed-model production systems is the appropriate utilization of available resources, including assembly workers. It should be noted that the organization of the linear production system imposes certain limitations. The assembly line consists of a given number of workstations, each of which determined by a space that it occupies. Assuming that the material handling system moves continuously at a predetermined speed, the station has only a limited time in which all the operations assigned to it must be finished. Limitation is mainly due to the physical constraints of stations space. Also in automatic lines robots, are able to perform only within strictly determined workspace. In the case of manual assembly lines, it is possible for the operator to cross the station borders, however due to the need of using tools located within the stations it is also limited. Moreover, this leads to unnecessary displacement and disorders in operators work at the next station.

With differences in labour-consumption between different products, producers may control the cycle time in order to ensure production flexibility. Cycle time may be elongated, and adapted to the product, execution of which requires the most effort (has the highest labour-consumption). However, this leads to idle times on workstations in a situation

![Figure 1. Evolution of linear assembly systems](image-url)
when a product with lower labour-consumption appears. On the other hand, shortening of the cycle time leads to frequent outages/stoppages of assembly line. As the producers usually must use the average cycle time, efforts on improving the assembly processes are rather focused on balancing lines and sequencing of production orders. The aim of determining the proper sequence is to ensure that the operator has the sufficient time to complete all assembly operations resulting from the planned production program and to provide a high level of utilization of workstations at the same time. This paper is a part of extensive work on tools for supporting production management in small and medium-sized enterprises conducted in the Institute of Automation Processes Engineering and Integrated Manufacturing Systems of the Silesian University of Technology and in the Department of Production Engineering of the University of Bialystok-Biala. Research focuses in particular on production organization integration, software development and technological processes planning [2, 14] as well as the product design, analysis and engineering calculations [12, 13].

2. Mixed-Model Assembly System

Mixed-model assembly systems are commonly found in the automotive industry, where at the same resources multiple products may be manufactured. Usually in such systems few different car models, with possibility of multiple engines, multiple colours of interior, multiple additional equipment components are produced in parallel with the production of special and limited versions as well. Usually level of production and the types of products are hardly predictive, thus the production plan may be subjected to constant changes (also as a result of distortions connected with parts availability in JIT systems). This type of environment requires a specific management, as such systems are supposed to achieve few different goals at the same time, usually denoted as the minimization of the completion time of a specific set of tasks, ensuring adequate saturation of assembly line as well as ensuring adequate utilization of human resources.

For the purpose of considered problem of determining a proper sequence of orders to be admitted to assembly, the actual production system in a car manufacturing company has been limited to a section located at the assembly department (Fig. 2). Considered system consists of an assembly line, into which production orders are distributed from a buffer [2]. Line consists of a given number of workstations. Few different products are assembled in the plant, each with a large number of versions. For the purpose of sequencing problem, based on current research on the nature of production and the quantities and types of products, manufactured versions were divided depending on the labour-consumption value into the classes from 1 to 7, where 7 indicates the most labour-consuming. Material handling system is realized through a chain-driven conveyor.

The conveyor moves at a constant speed through the whole assembly line, setting a steady rhythm of production. An additional element, which allows greater opportunities in determining the sequence of orders on the line, is the bypass – a part of conveyor line, into which the semi-product may be sent to, in the case it cannot be completed for various reasons. In the system two such places exist, first identified as Bypass 1 (long bypass), located on the exit of the buffer, but before entering the assembly lines. Second – Bypass 2, which begins with a line 8 in buffer (short bypass). Through both bypasses semi-product may be sent back to the beginning of the buffer. Buffer, out of which orders are transmitted into assembly line, consists of eight lines, limited by a certain capacity. In this case, only a buffer and a section of the conveyor just before and after it are locations where it is possible to perform proper sequencing of production orders. Each semi-product coming into the buffer has a specific identification code (ID). Orders selected for admission to assembly are released from the ends of various buffer lines. Also a special procedure for semi-products entering the buffer lines must be determined. Because only 8 buffer lines are available at the entrance to the buffer, it is not possible in the considered system to achieve a satisfactory sequence without earlier analyse of the orders at the entrance to the buffer lines.

![Image](image_url)

Figure 2. Simplified diagram of a conveyor lines on an assembly department
The car-bodies entering the buffer are allocated to the appropriate buffer line in such a way, that the result is a sequence, which in turn one after the other may be admitted into the assembly line. In this solution, appropriate sequencing of orders is taking part at the exit of the buffer, and also at its entrance. The lines are filled simultaneously, taking into account the previous car-body, which has been placed on them (Fig. 3). If there isn’t any option available to assign the car-body to a certain line, such a body would be directed to the line 8 and further to the bypass.

3. Order Sequencing in a Mixed Model Assembly System

Establishing an appropriate sequence of tasks consecutively admitted to be assembled in the manufacturing system is one of the major tasks in managing a mixed-model assembly systems. For the first time CSP (car sequencing problem) has been described in [4]. In [5] authors present general definition of an instance of the car sequencing problem. An instance of the problem is defined by a tuple \((V, o, p, q, r)\), where:

\[ V = \{v_1, ..., v_j\} \] is the set of vehicles to be produced;
\[ O = \{o_1, ..., o_j\} \] is the set of different options;
\[ p: O \rightarrow \infty \] and \[ q: O \rightarrow \infty \] define the capacity constraint associated with each option \(o \in O\), this constraint restricts, that for any subsequence of \(q_k\) consecutive cars on the line, no more than \(p_k\) of them may require option \(o_k\);
\[ r: V \times O \rightarrow \{0, 1\} \] defines the requirements for options, for example for each car \(v_j \in V\) and for each option \(o \in O, r_{ij} = 1\) if the option \(o\) must be installed in \(v_j\) and \(r_{ij} = 0\) otherwise.

It should be noted that two different cars of a set \(V\) may require the installation of the same configuration of option. All the vehicles that require installation of the same configuration options are grouped in the same class. More specifically, they form a \(k\) different vehicle classes, so that the set \(V\) is subdivided into \(k\) subsets \(V = V_1 \cup V_2 \cup \ldots \cup V_k\), such that all vehicles in a single subset \(V_j\) require the same configuration of options. Solution of the CSP problem is a set of cars in the sequence \(V\), thus defining the order in which they pass along the assembly line. CSP decision problem is to address the question if a sequence that satisfies all constraints imposed can be found, and optimization for CSP is to find a sequence that minimizes costs (the cost function should also take into account the non-fulfilment of constraints).

In its simplest form, the problem of car sequencing can also be formally written as in [6], i.e. an instance of a problem consists of \(n\) cars and \(m\) options. Each option represents the ratios of restrictions about \(p_k/q_k\) where \(p_k < q_k \leq n\). Each car \(c_i\) is defined as a bit string of length \(m\), such that \(c_{ij} = 1\) if the car and has the option of \(k\), and \(0\) otherwise. A solution is given as the sequence (which is a permutation) of \(n\) cars, such that, for each option \(o_k\), each subsequence \(q_k\) car has at most \(p_k\) cars equipped with the option \(o_k\).

4. Proposed method

The sequencing according to the basic CSP guidelines is possible only in systems with fixed production plans, and with no interferences in logistics processes. However, according to the authors, the modification of the approach proposed in the base version of the CSP problem can be considered, to match the current case [15]. Namely, production line should not be treated as a single resource for which the sequence is being determined, but as a set of workstations for which information is available about the influence caused by the different products in a sequence. This is caused by the fact that products may have different influence on certain workstations. Determining the sequence should take into account the workload of individual workstations rather than the entire line. This is done with the use of the vector of complexity, which can be described by a discrete function \(W_i\) for \(i = 1, ..., m\), where \(m\) is the number of machines:

\[
W_i = \begin{cases} 
W = 1 & \text{if product doesn’t affect the labour – consumption} \\
W > 1 & \text{otherwise} 
\end{cases}
\]

An example of the vector of complexity of the product 1 version 3 will be as follows:

\[
W_i = [1, 1, 1, 1, 1, 1, 1, 1, 2, 1, 1, 4, 4, 4, 4, 4, 4, 4, 4, 1, 1, 4, 1].
\]

Depending on the value of the vector of complexity labour-consumption class of the order is determined. Then, for a set of orders individual vectors of complexity are compared with each other and the sequence of tasks in order not to overload the system is determined [7]. The criterion for sequencing can take both the total number of overloading

Figure 3. Car bodies arranged into subsequences in the buffer lines
events in the system for a given sequence and the total time of overload caused by the specific sequence of tasks. The aim of the proposed method is to check how certain values change depending on the individual sequences, and to pick the most appropriate of them – the best fit according to the established criteria. Each production order appearing at the entrance to the buffer is analysed due to the vector of complexity. On the buffer lines, subsequences that facilitate proper sequencing of orders that are later on accepted into the assembly are being created. The initial creation of subsequences is not vital, however it improves sequencing orders at the end of the buffer.

At the output of the buffer a number of different sequences is being analysed. The optimization criteria were adopted to minimize the total time exceeded at the workstations – \( \text{min(}\text{SumPC}\text{)} \) and number of exceedances – \( \text{min(}\text{LPC}\text{)} \). The exceedance \( e_{l,i} \) of the \( i \)-th operation of the \( l \)-th order is calculated as:

\[
e_{l,i} = \max(0, t_{f_{l,i}} - t_{c_i} - c \cdot o_i)
\]

where:
- \( t_{f_{l,i}} \) – the finishing time of the \( i \)-th operation of the \( l \)-th order,
- \( t_{c_i} \) – the starting cycle time of \( i \)-th order (the first operation of the order),
- \( c \) – the working cycle of the system.

The \text{SumPC} performance measure is calculated by:

\[
\text{SumPC} = \sum_l \sum_o e_{l,o}
\]

The \text{LPC}_{l,o} performance measure of the \( o \)-th operation of the \( l \)-th order is calculated by:

\[
\text{LPC}_{l,o} = \begin{cases} 
1 & \text{if } e_{l,o} > 0 \\
0 & \text{otherwise}
\end{cases}
\]

The \text{LPC} performance measure for overall sequence of orders is calculated by:

\[
\text{LPC} = \sum_l \sum_o \text{LPC}_{l,o}
\]

Due to the nature of the system, the concurrent execution of operations is admissible. The employee may cross the border station. This however results in shorter time available for the execution of next production order on the workstation. It should also be noted that any overload (stations border crossing) adversely affects the system as it disturbs the rhythm in the next workstation.

5. The Prototype of Hybrid Solution for Tasks Sequencing

For the purposes of the problem of sequencing production orders in an automotive company a unique prototype of application, which, for a given set of production orders determines a sequence in which they can be adopted for assembly has been created.

Proposed hybrid approach is searching for a solution in parallel threads, with the use of implemented heuristics and meta-heuristics (e.g. Immune Algorithm [11]) and local search. The sequence of orders searching procedure is run periodically, at user-specified number of car-bodies admitted to the assembly. After each admission of certain number of orders for the assembly, the process of updating the priorities of the orders remaining in the buffer is conducted. It is a solution that prevents formation of blockages of car-bodies in the buffer. The time available to find a solution also allows the designation of one or more solutions. The conception of the functionality of the computer system is presented in [8] where the problem of scheduling in soft real-time conditions in manufacturing systems is discussed. The basic requirements of scheduling in a real time system and multi thread approach with rule-based heuristics, meta-heuristics and random searching is presented there.

The selection of solutions is carried out in two stages, using a multi-criterion evaluation. For practical reasons, weighted sum method has been applied, to provide the ability to use, in addition to the previously described basic criteria, other criteria defined by the decision makers. In [9, 10] the multicriteria optimization problem in scheduling of manufacturing systems is widely presented. Described decision-making stages that should be taken in scheduling process became the basis of the application development. In the first stage, each of the algorithms evaluates designated solutions and selects a specified number of the best of them. These solutions subsequently enter the common list. From such created ranking list, solution to be implemented is being chosen.

To start operation of the order sequencing software all the required data, e.g. the number of workstations, processes and operations in processes (including the determination of the scope for concurrent execution of the operations on the following workstations), should be introduced in the appropriate forms.

After entering the data, searching for the proper sequence can be started. The program begins to check different sequences and calculates parameters defined by user in a real time (Fig. 4). In the present case, the criteria were the number of exceeded cycles (MZ_LPC) and the total time exceeded in workstations (MZ_SumPC). Depending on the pre-set stop condition, the program may terminate its operation after a predetermined number of iterations, or with the approval of the selected indicators.

While searching the solution space achieved results are also presented. Presentation is useful in so, that the user can observe how the adopted optimization criteria change, and basing on that information to stop the calculation at any time. After stopping the program, results can be viewed and the decision upon which sequence achieved the highest point value of the objective function can be made. The results may be sorted, which greatly speeds up the process of searching for desired solutions. (Fig. 5). In the present case, the criterion was the number of exceeded cycles (MZ_LPC) and the total time exceeded in workstations (MZ_SumPC). In addition, the work schedule, which shows the flow of the order through the assembly line (Fig. 6) is generated. The further of the work on the program is planned to create a tool for scheduling workers activities, which in the case of large cycle exceeds will allow to introduce additional staff (floaters), that will be responsible for assisting in the execution of tasks that exceed the cycle time of the workstations.

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Figure 4. Sequencing process in the hybrid sequence-generation environment

Figure 5. Searching the solution space (common list visualisation)

Figure 6. Results
6. Summary

This paper presents the issues related to the mixed-model production in linear production systems. Problems associated with the presented class of production systems, and the reasons why sequencing of orders is vital in terms of the systems proper functioning have been discussed. Authors present the essence of the CSP problem on the example of car manufacturer. The genesis of the problem and a detailed description of the problem instance is presented. General description of proposed method is given. The functioning of a hybrid computer sequencing system containing heuristics and meta-heuristics (with the possibility to implement further algorithms), created for the purpose of sequencing orders in the considered company is described. In presented solution the minimization of two basic measures is used: the total time exceeded at the workstations – min(SumPC) and number of exceedances – min(LPC). Practical determination of the assessment should be based on a larger number of criteria, apart from the above, also i.e. the average time or standard deviation of exceedances. Adapted multi-criteria evaluation method provides the ability to conduct searches in various forms, both „a priori” and „a posteriori” [10]. Interactive variant of evaluation process is allowed only when the time to search for a solution can be extended for the next cycle of calculation. Further work on the subject should also concern on searching for other algorithms, which may be implemented into the computer system in order to improve the speed of calculation processes and quality of the solutions obtained.

References:


**Key words:**
mixed-model assembly, CSP, sequencing.

**Abstract:**
In the paper issues related to the mixed-model production in linear systems on the example of a car manufacturing company are discussed. The origins of linear assembly systems are described, and problems concerning determining the sequence of tasks to be admitted to assembly processes are pointed out. The computer system for projecting the production orders sequence in a mixed-model production system, based on a hybrid solution is presented. Also basics of the CSP (car sequencing problem) are discussed.

**KOMPUTEROWO WSPOMAGANE SEKWENCJONOWANIE ZLECEN W SYSTEMIE PRODUKCJI MIXED-MODEL**

**Słowa kluczowe:**
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